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Frequency Domain Equalization of DMT/OFDM Systems with Insufficient Guard Interval

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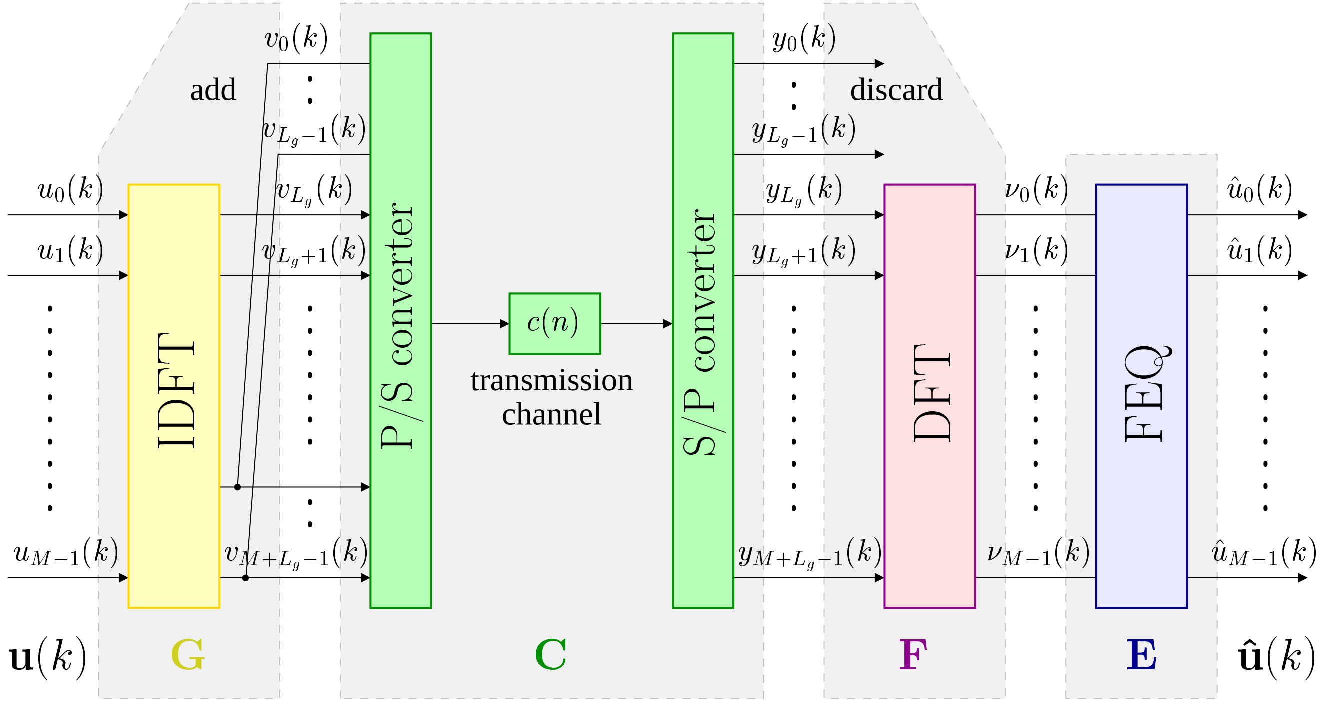
Overview

- Introduction & Motivation
- Basic Concept of the New Algorithm
- Key Features
- Application
- Conclusions

Introduction

- **DMT** (Discrete Multi-Tone) = **OFDM** (Orthogonal Frequency Division Multiplexing)



- transfer function in polyphase and block matrix notation

$$\hat{\mathbf{U}}(z) = \mathbf{E}(z) \cdot \mathbf{F}(z) \cdot z^{-1} \cdot \mathbf{C}(z) \cdot \mathbf{G}(z) \cdot \mathbf{U}(z)$$

$$\bullet\!\!-\!\!\circ$$

$$\hat{\mathbf{u}}(k) = \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C} \cdot \mathbf{G} \cdot \mathbf{u}^{(n)}(k-1)$$

(n – number of interfering symbols)

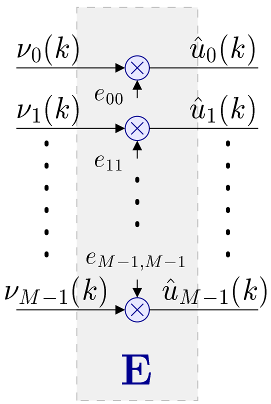
Importance of the Guard Interval

$$\hat{\mathbf{u}}(k) = \underbrace{\mathbf{E} \cdot \overbrace{\mathbf{F} \cdot \mathbf{C} \cdot \mathbf{G}}^{\mathbf{H}}}_{\mathbf{T}} \cdot \mathbf{u}^{(n)}(k-1)$$

Sufficient GI: $L_g \geq L_c - 1$

- $n = 1 \longrightarrow$ no ISI
- cyclic prefix $\longrightarrow$ sequential convolution with channel impulse response becomes virtually cyclic $\longrightarrow$ no ICI

$$\mathbf{T} = \mathbf{E} \cdot \mathbf{H} = \mathbf{E} \cdot \mathbf{D} \stackrel{!}{=} \mathbf{I}$$



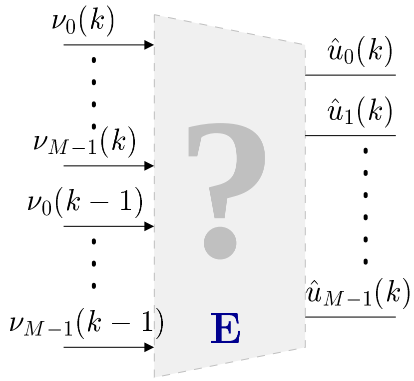
- $\mathbf{D}$ diagonal matrix, guaranteed solution $\mathbf{E} = \mathbf{D}^{-1}$ with $e_{ii} = 1/C(\mathrm{e}^{j2\pi i/M})$
- Efficient equalization with only one complex multiplication per carrier

Insufficient GI: $L_g < L_c - 1$

- $n > 1 \longrightarrow$ (severe) ISI
- no longer cyclic conv. $\longrightarrow$ (severe) ICI

$$\mathbf{T} = \mathbf{E} \cdot \mathbf{H} \stackrel{!}{=} [\underbrace{\mathbf{0} \ \cdots \ \mathbf{0} \ \mathbf{I} \ \mathbf{0} \ \cdots \ \mathbf{0}}_{n}]$$

- solution only if $\mathbf{E}$ takes neighboring symbols into account
- $\mathbf{E}$ very big and no sparse structure



Drawbacks of the Guard Interval

- Currently, most DMT/OFDM applications use a Guard Interval and try to achieve $L_g \geq L_c - 1$, but:

  $\rightarrow$ **Bandwidth efficiency problem:** With increasing L_g, bandwidth efficiency reduces by factor $M/(M + L_g)$ $\rightarrow$ supporting techniques like time-domain equalizer (TEQ) shorten the effective length of the channel impulse response $\rightarrow$ guard interval of certain length still required

  $\rightarrow$ **Redundancy type problem:** While guard interval introduces extra redundancy in time domain, existing redundancy in frequency domain remains unused

  $\rightarrow$ **Optimization problem:** Optimization of the TEQ coefficients in time-domain has no direct relation to the actual design goal, the channel throughput

Question:

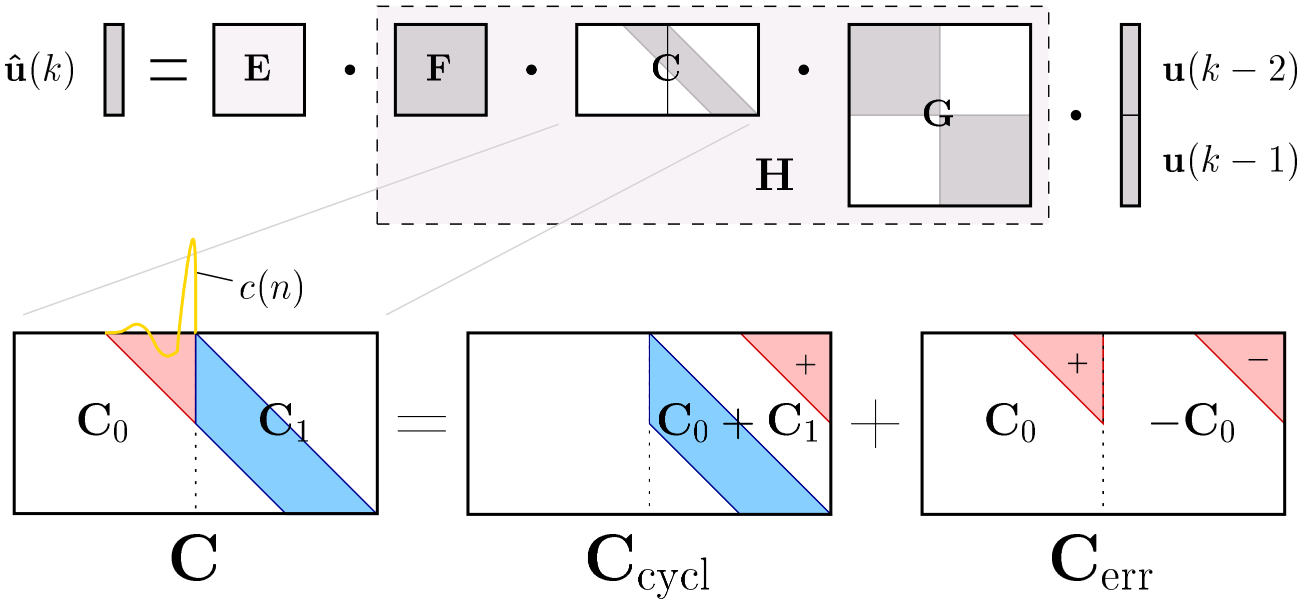
$\Rightarrow$ Is it possible to define a more general DMT/OFDM scheme, which

- provides higher bandwidth efficiency by taking existing frequency domain redundancy into account,
- allows for perfect equalization in the case of insufficient or even non-existing guard interval,
- compensates for ISI/ICI while maintaining symbol-separate equalization,
- shows higher robustness to a violation of the perfect-equalization condition,
- is in most parts identical to the DMT/OFDM structure and does not increase the overall complexity,
- allows the usage of simple adaptation methods like LMS or RLS?

$\Rightarrow$ **Answer: YES!**

Basic Idea #1: Extraction of ISI/ICI-Generating Part in Channel Matrix C

- Assumption: no Guard Interval, $L_c \leq M$, no pre-cursor $\rightarrow$ ISI from the preceding symbol



$\Rightarrow$ $\mathbf{C}$ can be split into ideal, cyclic part $\mathbf{C}_{\text{cycl}}$ and ISI/ICI error part $\mathbf{C}_{\text{err}}$

$$\mathbf{C} = [\mathbf{C}_0 \quad \mathbf{C}_1] = \underbrace{[\mathbf{0}_M \quad (\mathbf{C}_0 + \mathbf{C}_1)]}_{\mathbf{C}_{\text{cycl}}} + \underbrace{[\mathbf{C}_0 \quad (-\mathbf{C}_0)]}_{\mathbf{C}_{\text{err}}}$$

Extraction of ISI/ICI-Part in Channel Matrix $\mathbf{C}$ (Cont'd)

⇒ Substitution into transfer function

$$\mathbf{E} \cdot \mathbf{H} = \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C} \cdot \mathbf{G} = \mathbf{E} \cdot \mathbf{F} \cdot (\mathbf{C}_{\text{cycl}} + \mathbf{C}_{\text{err}}) \cdot \mathbf{G} \stackrel{!}{=} [\mathbf{0} \quad \mathbf{I}]$$

⇒ With $\mathbf{C}_{\text{cycl,red}} = \mathbf{C}_0 + \mathbf{C}_1$ and $\mathbf{G}'$ as a diagonal block in $\mathbf{G}$, separation into two sub-systems possible

$$\textbf{I}: \quad \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C}_{\text{cycl,red}} \cdot \mathbf{G}' \stackrel{!}{=} \mathbf{I}$$

$$\textbf{II}: \quad \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C}_{\text{err}} \cdot \mathbf{G} \stackrel{!}{=} [\mathbf{0} \quad \mathbf{0}]$$

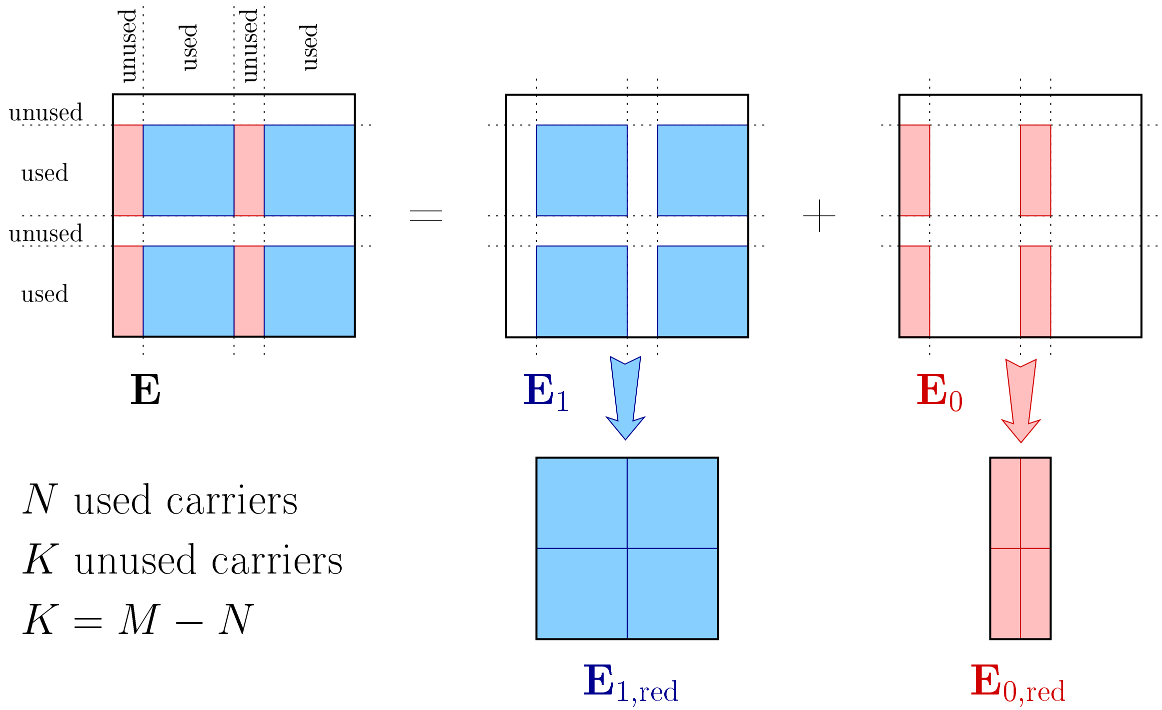
→ Equation system **I** describes an ideal, distortion-free DMT system

→ Equation system **II** eliminates ISI/ICI caused by $\mathbf{C}_{\text{err}}$

Basic Idea #2: Decomposition of Equalizer Matrix $\mathbb{E}$ in Case of Unused Carriers

- Under assumption that K carriers not used decomposition of equalizer matrix into

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_0$$



N used carriers
K unused carriers
$K = M - N$

$\rightarrow$ $\mathbf{E}_0$ contains K columns of unused carrier output samples

$\rightarrow$ $\mathbf{E}_1$ contains $N = M - K$ columns of used carrier output samples

Perfect Equalization Condition

⇒ After elimination of zero rows and columns equation system **I** independent from $\mathbf{E}_0$

$$\mathbf{E}_{1,\text{red}} \cdot \underbrace{\mathbf{F}_{1,\text{red}} \cdot \mathbf{C}_{\text{cycl,red}} \cdot \mathbf{G}'_{\text{red}}}_{\mathbf{D}_{\text{red}}} \stackrel{!}{=} \mathbf{I}_N$$

- Guaranteed solution for **I** with $\mathbf{E}_{1,\text{red}} = \mathbf{D}_{\text{red}}^{-1}$ → deg. of freedom in $\mathbf{E}_{0,\text{red}}$ used for solving **II** !

⇒ Solution independent from channel frequency response at the unused carrier positions!

- Solution for **II** exists if

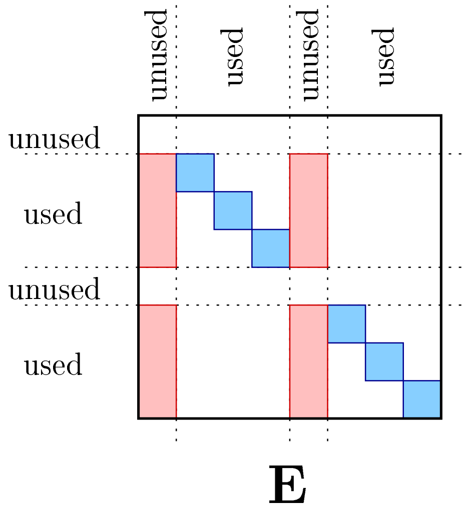
$$K + L_g \geq L_c - 1$$

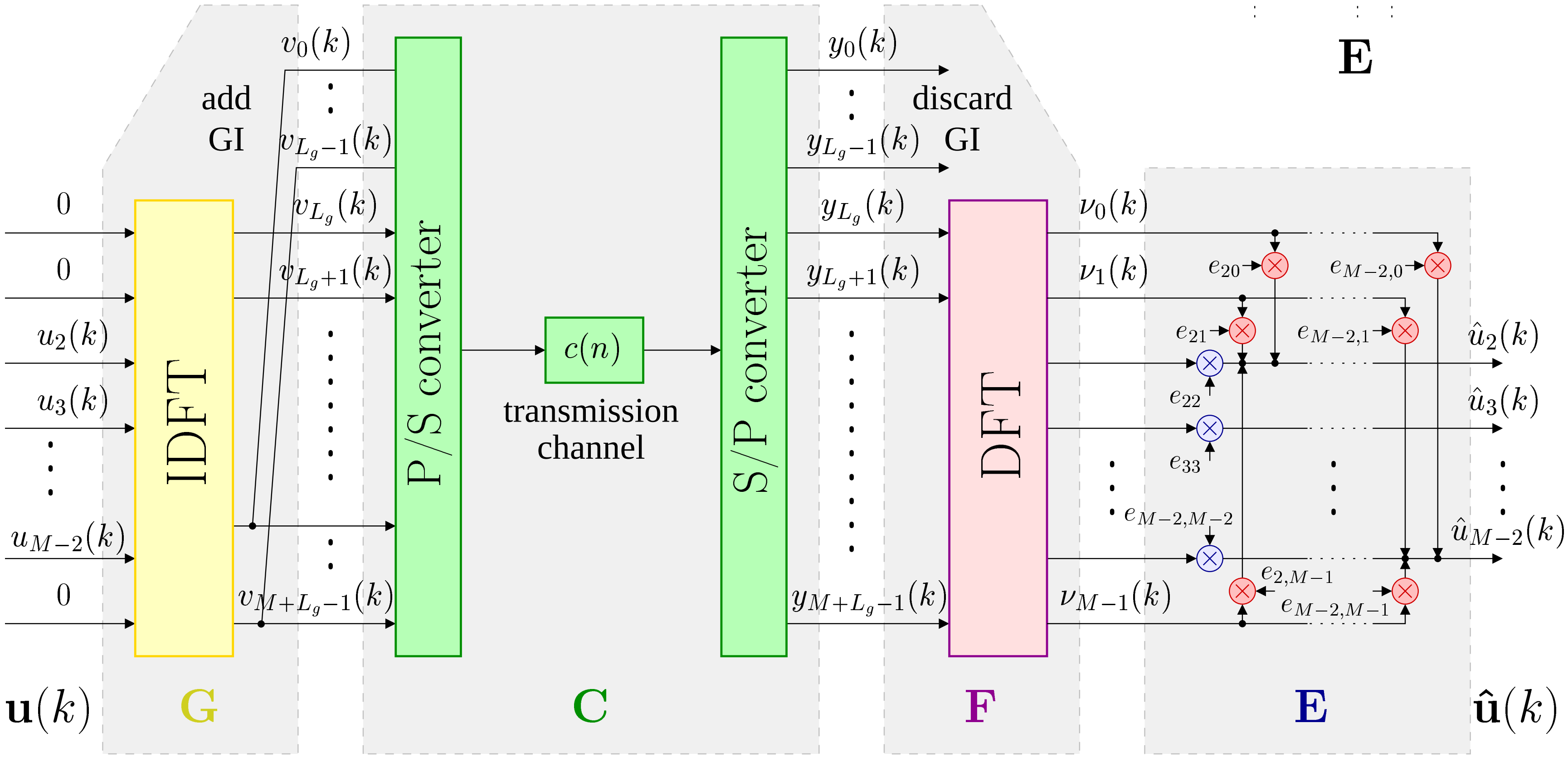
⇒ Ideal combination of TD and FD redundancy → Arbitrary distribution

- Special cases:
  → $K = 0$: Traditional DMT/OFDM without usage of FD redundancy
  → $L_g = 0$: No guard interval, but symbol-separate, perfect equalization!

Proposed Structure ⇒ Generalized DMT/OFDM

- Transmitter remains unchanged
- Receiver with extended FEQ → Sparse Equalizer Matrix $\mathbf{E}$

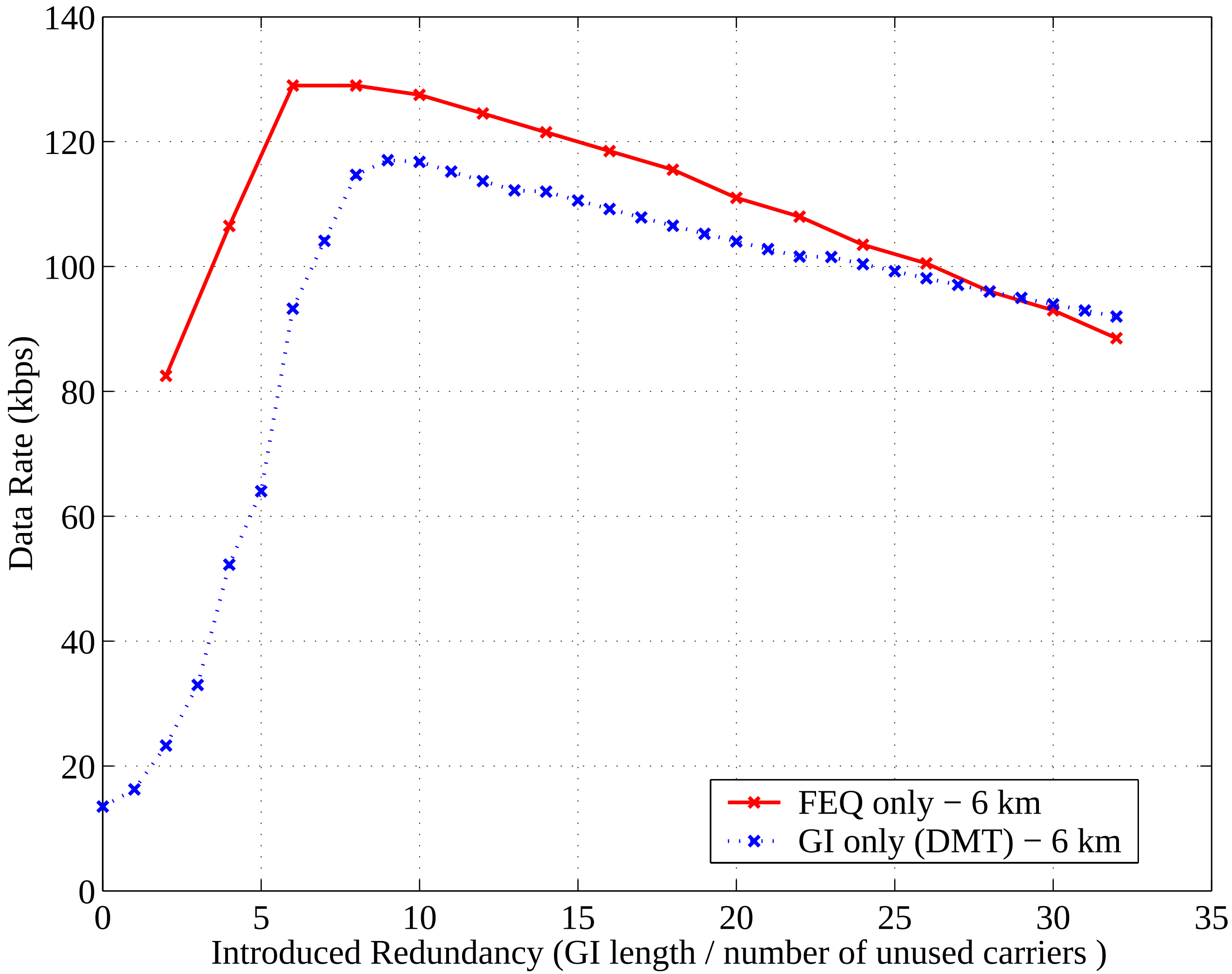




GDMT/GOFDM Key Features

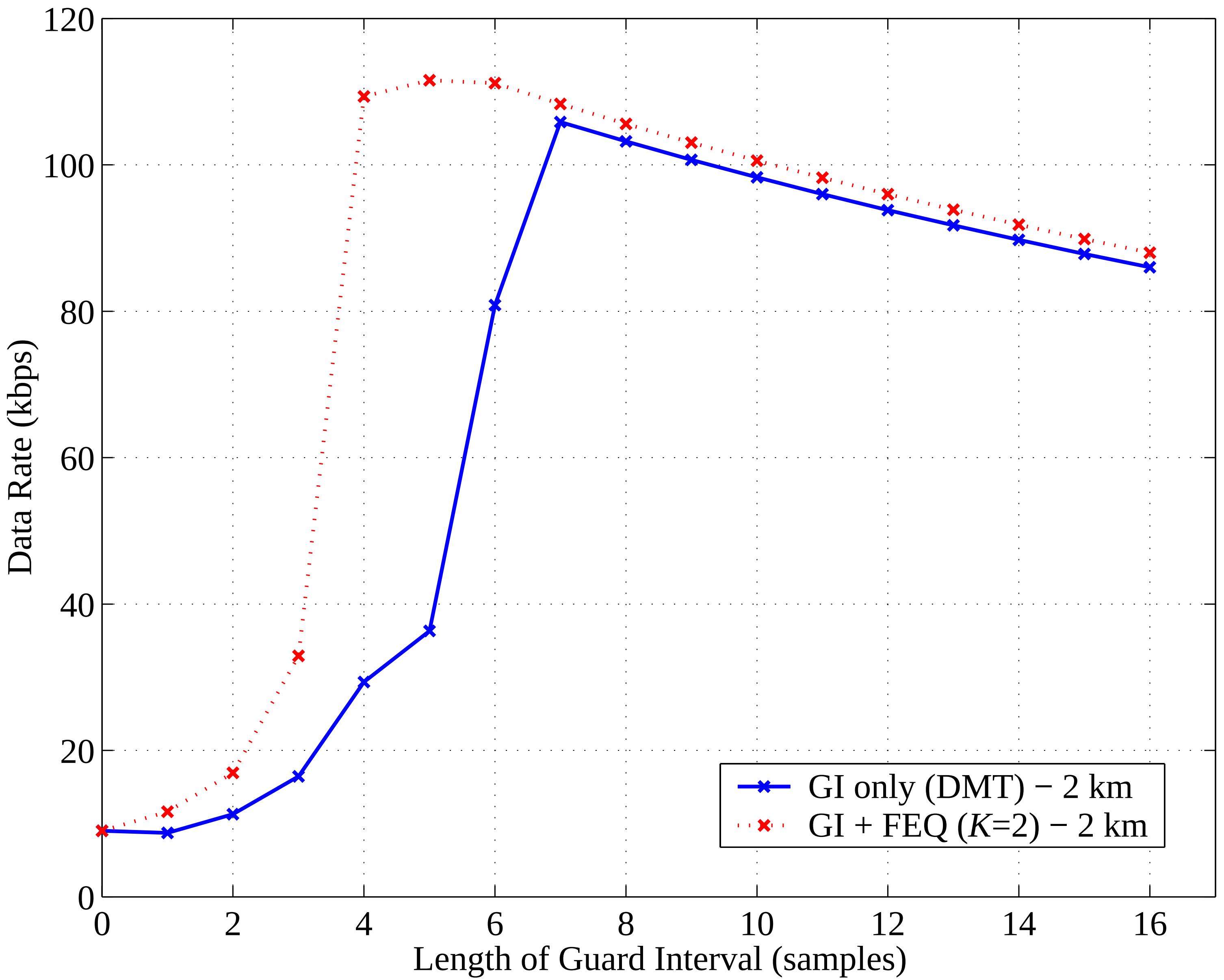
Special Case: No Guard Interval ($L_g = 0$)

(MatLab simulation of a proprietary DMT system with $f_s = 48$ kHz, $B = 21$ kHz, AWGN SNR=30 dB, twisted-pair loops with $l = 2$ to 30 km, $d = 0.9$ mm, DFT length $M = 64$)



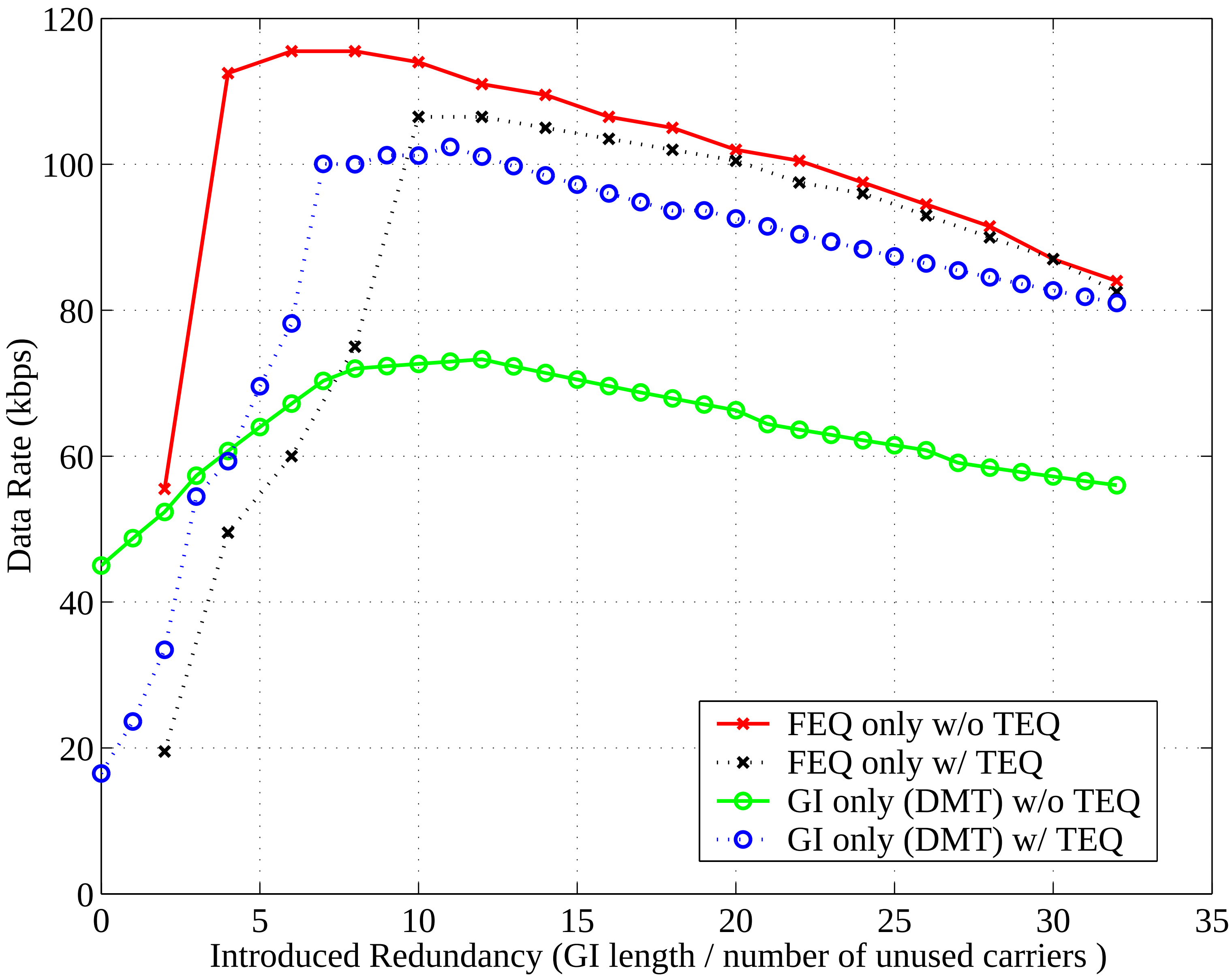
Higher Bandwidth Efficiency

($M = 64$, only the two unusable carriers at DC and $f_s/2$ utilized for the FEQ)



'Built-in' Capability for Shortening the Channel Impulse Response

(10km loop, 244 coeffs. without TEQ, 13 effective coeffs. after TEQ, $M = 64$)



GDMT/GOFDM Application

⇒ Implementation complexity of GDMT/GOFDM for typical constellations equal or even smaller than DMT+TEQ combination!

- **Standard-compliant**
  - GDMT/GOFDM can be implemented in the receiver without knowledge of the transmitter → Only precondition: a certain number of unused carriers
  - Works as a virtual extension of the guard interval → improved receiver in terms of data rate, range, ISI/ICI & RFI robustness and/or system latency

- **Non- or future standard**
  - Most flexible application of GDMT/GOFDM → Allows for arbitrary distribution of the necessary redundancy, either to time or frequency domain, depending on the respective design criteria
  - Includes the special without guard interval → minimum system latency time

Concluding Remarks

- Additionally to Guard Interval insertion, GDMT/GOFDM incorporates frequency domain redundancy by extending the DMT single-tap equalizer to a block equalizer → Traditional DMT/OFDM as a special case
- Given sufficient conditions for perfect equalization the only non-zero entries in the block equalizer are the ones that are already present in conventional DMT plus additional branches that combine used subcarriers with unused subcarriers
- Channel characteristics at the position of the carriers used for the extended FEQ is of no importance → New FEQ scheme only utilizes ISI/ICI leakage caused by the FFT operation at the receiver side → Especially strong and wide-spread for DMT → Poor spectral selectivity of the DFT basis filters becomes an advantage!
- GDMT/GOFDM not restricted to a special physical channel → Can be applied to existing or future standards in wired communications (ADSL, VDSL, Powerline, etc.), or mobile communications (DAB, DVB, HiperLAN/2, etc.)